EMBEDDING
FOUR-DIRECTIONAL PATHS
ON CONVEX POINT SETS

Oswin Aichholzer
Thomas Hackl
Birgit Vogtenhuber

Sarah Lutteropp
Tamara Mchedlidze
Embedding 4-directional path on a convex point set
Embedding 4-directional path on a convex point set
Embedding 4-directional path on a convex point set
MOTIVATION & PREVIOUS WORK

KNOWN RESULTS

Always possible for $\leq 10$

Directed order types

Embedding 4-directional path on a convex point set
MOTIVATION & PREVIOUS WORK

ORIENTED PATH  POINT SET  UPWARD PLANAR EMBEDDING

KNOWN RESULTS

Always possible for $\leq 10$

Directed order types
Binucci et al. CGTA10
Angelini el al. GD10

Several special cases of paths
MOTIVATION & PREVIOUS WORK

ORIENTED PATH

POINT SET

UPWARD PLANAR EMBEDDING

KNOWN RESULTS

Always possible for $\leq 10$

Directed order types

Binucci et al. CGTA10

Angelini el al. GD10

Convex point sets

Binucci et al. CGTA10

Embedding 4-directional path on a convex point set
Embedding 4-directional path on a convex point set
MOTIVATION & PREVIOUS WORK

ORIENTED PATH

POINT SET

UPWARD PLANAR EMBEDDING

QUESTION

Is it possible for any point set in general position?
MOTIVATION & PREVIOUS WORK

ORIENTED PATH  POINT SET  UPWARD PLANAR EMBEDDING

QUESTION

Is it possible for any point set in general position?

We still do not know 😞
LET’S LOOK AT NUMBERS
LET’S LOOK AT NUMBERS

How many distinct plane spanning paths has a point set?
LET’S LOOK AT NUMBERS

How many distinct plane spanning paths has a point set?
LET’S LOOK AT NUMBERS

How many distinct plane spanning paths has a point set?
LET’S LOOK AT NUMBERS

How many distinct plane spanning paths has a point set?
LET’S LOOK AT NUMBERS

How many distinct plane spanning paths has a point set?

At least $n2^{n-3}$, which is achieved by convex point sets
LET’S LOOK AT NUMBERS

How many distinct plane spanning paths has a point set?

At least $n2^{n-3}$, which is achieved by convex point sets

How many oriented paths exist?
LET’S LOOK AT NUMBERS

How many distinct plane spanning paths has a point set?
At least $n2^{n-3}$, which is achieved by convex point sets

How many oriented paths exist?
There are $2^{n-2}$ oriented paths

Even for convex point sets it is surprising that embedding always exists
LET’S LOOK AT NUMBERS

How many distinct plane spanning paths has a point set?
At least $n2^{n-3}$, which is achieved by convex point sets

How many oriented paths exist?
There are $2^{n-2}$ oriented paths

Even for convex point sets it is surprising that embedding always exists

Different view on an oriented path
LET’S LOOK AT NUMBERS

How many distinct plane spanning paths has a point set?
At least $n2^{n-3}$, which is achieved by convex point sets

How many oriented paths exist?
There are $2^{n-2}$ oriented paths

Even for convex point sets it is surprising that embedding always exists

Different view on an oriented path

Embedding 4-directional path on a convex point set
LETS LOOK AT NUMBERS

How many distinct plane spanning paths has a point set?
At least $n2^n - 3$, which is achieved by convex point sets

How many oriented paths exist?
There are $2^{n-2}$ oriented paths

Even for convex point sets it is surprising that embedding always exists

Different view on an oriented path

| U | U | D | U | D |

Embedding 4-directional path on a convex point set
**LET’S LOOK AT NUMBERS**

How many distinct plane spanning paths has a point set?

At least $n2^{n-3}$, which is achieved by convex point sets

How many oriented paths exist?

There are $2^{n-2}$ oriented paths

Even for convex point sets it is surprising that embedding always exists

Different view on an oriented path

```
U U D U D
```
PROBLEM DEFINITION & RESULTS

FOR CONVEX POINT SETS THE NUMBERS ARE TIGHT

WHAT HAPPENS WITH FOUR?
FOR CONVEX POINT SETS THE NUMBERS ARE TIGHT
WHAT HAPPENS WITH FOUR?

PROBLEM DEFINITION
PROBLEM DEFINITION & RESULTS

FOR CONVEX POINT SETS THE NUMBERS ARE TIGHT
WHAT HAPPENS WITH FOUR?

PROBLEM DEFINITION

Embedding 4-directional path on a convex point set
FOR CONVEX POINT SETS THE NUMBERS ARE TIGHT
WHAT HAPPENS WITH FOUR?

PROBLEM DEFINITION

Embedding 4-directional path on a convex point set
FOR CONVEX POINT SETS THE NUMBERS ARE TIGHT
WHAT HAPPENS WITH FOUR?

PROBLEM DEFINITION

DIRECTION-CONSISTENT EMBEDDING
PROBLEM DEFINITION & RESULTS

For convex point sets the numbers are tight.

What happens with four?

**Problem Definition & Results**

<table>
<thead>
<tr>
<th>U</th>
<th>R</th>
<th>L</th>
<th>D</th>
</tr>
</thead>
</table>

Direction-consistent embedding

Embedding 4-directional path on a convex point set
FOR CONVEX POINT SETS THE NUMBERS ARE TIGHT
WHAT HAPPENS WITH FOUR?

PROBLEM DEFINITION & RESULTS

U R L D

DIRECTION-CONSISTENT EMBEDDING

RESULTS
Not always possible for four directions
**PROBLEM DEFINITION**

<table>
<thead>
<tr>
<th>U</th>
<th>R</th>
<th>L</th>
<th>D</th>
</tr>
</thead>
</table>

**RESULTS**

- Not always possible for four directions
- Always possible for three directions

**DIRECTION-CONSISTENT EMBEDDING**
FOR CONVEX POINT SETS THE NUMBERS ARE TIGHT
WHAT HAPPENS WITH FOUR?

PROBLEM DEFINITION & RESULTS

Not always possible for four directions
Always possible for three directions
Can be decided in $O(n^2)$ time for four directions.
There are $n2^{n-3}$ oriented paths.

Each can be labeled in $2^{n-1}$ ways and read from 2 end-vertices.

In total at most $n2^{2n-3}$ plane 4-directional paths on a convex point set.

To compare with $2^{2n-2}$ 4-directional paths.
There are $n2^{n-3}$ oriented paths.

Each can be labeled in $2^{n-1}$ ways and read from 2 end-vertices.

In total at most $n2^{2n-3}$ plane 4-directional paths on a convex point set.

To compare with $2^{2n-2}$ 4-directional paths.
There are $n2^{n-3}$ oriented paths. Each can be labeled in $2^{n-1}$ ways and read from 2 end-vertices. In total at most $n2^{2n-3}$ plane 4-directional paths on a convex point set. To compare with $2^{2n-2}$ 4-directional paths.
There are $n2^{n-3}$ oriented paths.

Each can be labeled in $2^{n-1}$ ways and read from 2 end-vertices.

In total at most $n2^{2n-3}$ plane 4-directional paths on a convex point set.

To compare with $2^{2n-2}$ 4-directional paths.
There are $n2^{n-3}$ oriented paths.

Each can be labeled in $2^{n-1}$ ways and read from 2 end-vertices.

In total at most $n2^{2n-3}$ plane 4-directional paths on a convex point set.

To compare with $2^{2n-2}$ 4-directional paths.

Embedding 4-directional path on a convex point set.
There are $n2^{n-3}$ oriented paths.

Each can be labeled in $2^{n-1}$ ways and read from 2 end-vertices.

In total at most $n2^{2n-3}$ plane 4-directional paths on a convex point set.

To compare with $2^{2n-2}$ 4-directional paths.
COUNTING?

There are $n2^{n-3}$ oriented paths.

Each can be labeled in $2^{n-1}$ ways and read from 2 end-vertices.

In total at most $n2^{2n-3}$ plane 4-directional paths on a convex point set.

To compare with $2^{2n-2}$ 4-directional paths.

Embedding 4-directional path on a convex point set.
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set
**THEOREM**
Any three-directional path admits a direction-consistent embedding on any convex point set

**“ONE-SIDED” LEMMA**
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“ONE-SIDED” LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set.

“ONE-SIDED” LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set.
**THEOREM**
Any three-directional path admits a direction-consistent embedding on any convex point set.

**“ONE-SIDED” LEMMA**
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set.

**“PROOF”**
Proceed the path backward. Choose the topmost (bottomost, rightmost) free point, if the previous edge has label U (D,R).
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set.

“ONE-SIDED” LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set.

“PROOF”
Proceed the path backward. Choose the topmost (bottomost, rightmost) free point, if the previous edge has label U (D,R).
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“ONE-SIDED” LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set

“PROOF”
Proceed the path backward. Choose the topmost (bottomost, rightmost) free point, if the previous edge has label U (D,R).
THREE-DIRECTIONAL 😊

THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set.

“ONE-SIDED” LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set.

“PROOF”
Proceed the path backward. Choose the topmost (bottommost, rightmost) free point, if the previous edge has label U (D,R).
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set.

“ONE-SIDED” LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set.

“PROOF”
Proceed the path backward. Choose the topmost (bottommost, rightmost) free point, if the previous edge has label U (D,R).
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“ONE-SIDED” LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set

“PROOF”
Proceed the path backward. Choose the topmost (bottommost, rightmost) free point, if the previous edge has label U (D,R).
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

"ONE-SIDED" LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set

"PROOF"
Proceed the path backward. Choose the topmost (bottomost, rightmost) free point, if the previous edge has label U (D,R).
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set.

“ONE-SIDED” LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set.

“PROOF”
Proceed the path backward. Choose the topmost (bottommost, rightmost) free point, if the previous edge has label U (D,R).
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set.

“ONE-SIDED” LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set.

“PROOF”
Proceed the path backward. Choose the topmost (bottommost, rightmost) free point, if the previous edge has label U (D,R).
**THEOREM**

Any three-directional path admits a direction-consistent embedding on any convex point set.

**“ONE-SIDED” LEMMA**

A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set.

**“PROOF”**

Proceed the path backward. Choose the topmost (bottomost, rightmost) free point, if the previous edge has label U (D,R).
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“ONE-SIDED” LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set

“PROOF”
Proceed the path backward. Choose the topmost (bottommost, rightmost) free point, if the previous edge has label U (D,R).
THREE-DIRECTIONAL 😊

THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“ONE-SIDED” LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a one-sided convex point set

“PROOF”
Proceed the path backward. Choose the topmost (bottommost, rightmost) free point, if the previous edge has label U (D,R).
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“STRIP-CONVEX” LEMMA
A \{U,R\}-path admits a direction-consistent embedding on a strip-convex point set
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“STRIP-CONVEX” LEMMA
A \{U,R\}-path admits a direction-consistent embedding on a strip-convex point set
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“STRIP-CONVEX” LEMMA
A \{U,R\}-path admits a direction-consistent embedding on a strip-convex point set
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“STRIP-CONVEX” LEMMA
A \{U,R\}-path admits a direction-consistent embedding on a strip-convex point set
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“STRIP-CONVEX” LEMMA
A \{U,R\}-path admits a direction-consistent embedding on a strip-convex point set

“PROOF”
Apply the same algorithm. Observe that the identified points are consecutive.
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“STRIP-CONVEX” LEMMA
A \{U,R\}-path admits a direction-consistent embedding on a strip-convex point set

“PROOF”
Apply the same algorithm. Observe that the identified points are consecutive.
THEOREM

Any three-directional path admits a direction-consistent embedding on any convex point set.
{U,D,R}-LEMMA
A {U,D,R}-path admits a direction-consistent embedding on a convex point set*
\{U,D,R\}-LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
\{U,D,R\}-LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
\{U,D,R\}-LEMMA

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
\{U,D,R\}-LEMMA

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
**U,D,R**-LEMMA

A **U,D,R**-path admits a direction-consistent embedding on a convex point set*

“PROOF”
**{U,D,R}-LEMMA**
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“**PROOF**”
One of the boundary edges is D
**{U,D,R}-LEMMA**

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

**“PROOF”**

One of the boundary edges is D
\{U,D,R\}-LEMMA

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”

One of the boundary edges is D

Apply “one-sided” Lemma
\{U,D,R\}-LEMMA

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”

One of the boundary edges is D

Apply “one-sided” Lemma
\{U,D,R\}-LEMMMA

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”

One of the boundary edges is D

Apply “one-sided” Lemma
\{U,D,R\}-LEMMA

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”

One of the boundary edges is D

Apply “one-sided” Lemma
**U,D,R**-LEMMA

A **U,D,R**-path admits a direction-consistent embedding on a convex point set*

**“PROOF”**

One of the boundary edges is **D**

Apply “one-sided” Lemma
{U,D,R}-LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
One of the boundary edges is D
Apply “one-sided” Lemma
\{U,D,R\}-LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
One of the boundary edges is D
Apply “one-sided” Lemma
\{U,D,R\}-LEMMA

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”

One of the boundary edges is D

Apply “one-sided” Lemma

Embedding 4-directional path on a convex point set
\{U,D,R\}-LEMMMA

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”

One of the boundary edges is \textbf{D}

Apply “one-sided” Lemma

Both boundary edges are \textbf{D}
\{U,D,R\}-LEMMA
A $\{U,D,R\}$-path admits a direction-consistent embedding on a convex point set*

“PROOF”
One of the boundary edges is D
Apply “one-sided” Lemma
Both boundary edges are D

Embedding 4-directional path on a convex point set
**THREE-DIRECTIONAL 😊**

**{U,D,R}-LEMMA**
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

**“PROOF”**
One of the boundary edges is D
Apply “one-sided” Lemma
Both boundary edges are D
\textbf{\{U,D,R\}-LEMMA}

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

\textbf{“PROOF”}

One of the boundary edges is \textbf{D}

Apply “one-sided” Lemma

Both boundary edges are \textbf{D}

Apply “one-sided” Lemma
**{U,D,R}-LEMMA**

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

**“PROOF”**

One of the boundary edges is \textbf{D}

Apply “one-sided” Lemma

Both boundary edges are \textbf{D}

Apply “one-sided” Lemma

---

*Note: The asterisk (*) indicates a footnote or reference that is not provided in the image.
\{U,D,R\}-LEMMA

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”

One of the boundary edges is D

Apply “one-sided” Lemma

Both boundary edges are D

Apply “one-sided” Lemma
**{U,D,R}-Lemma**

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

**“Proof”**

One of the boundary edges is D

Apply “one-sided” Lemma

Both boundary edges are D

Apply “one-sided” Lemma

Sort by y-coordinate
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

"PROOF"
One of the boundary edges is D
Apply "one-sided" Lemma
Both boundary edges are D
Apply "one-sided" Lemma
Sort by y-coordinate
\{U,D,R\}-LEMMA

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
\{U,D,R\}\-LEMMA

A \{U,D,R\}\-path admits a direction-consistent embedding on a convex point set*

“PROOF”

Both boundary edges are U/R
\textbf{\{U,D,R\}-LEMMA}

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

\textbf{“PROOF”}

Both boundary edges are U/R
\textbf{\{U,D,R\}-LEMMA}

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

"PROOF"

Both boundary edges are U/R
**\{U,D,R\}-LEMMA**

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

**“PROOF”**

Both boundary edges are U/R
**THREE-DIRECTIONAL**

\{U,D,R\}-**LEMMA**

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

**“PROOF”**

Both boundary edges are **U/R**
\{U,D,R\}-LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
Both boundary edges are \(U/R\)
None fit - One fit - Both fit
\{U,D,R\}\text{-}\text{LEMMA}

A \{U,D,R\}\text{-}path admits a direction-consistent embedding on a convex point set*

“PROOF”

Both boundary edges are U/R

None fit - One fit - Both fit
**{U,D,R}-LEMMA**

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

**“PROOF”**

Both boundary edges are \textbf{U/R}

None fit - One fit - Both fit

Apply “one-sided” Lemma
\{U,D,R\}-LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
Both boundary edges are U/R
None fit - One fit - Both fit
Apply “one-sided” Lemma
\{U,D,R\}-LEMMA

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”

Both boundary edges are U/R

None fit - One fit - Both fit

Apply “one-sided” Lemma
**{U,D,R}-LEMMA**

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

**“PROOF”**

Both boundary edges are U/R

None fit - One fit - Both fit

Apply “one-sided” Lemma
Apply “strip-convex” Lemma
THE THREE-DIRECTIONAL 😊

**{U,D,R}-LEMMA**
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
Both boundary edges are U/R
None fit - One fit - Both fit
{U,D,R}-LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
Both boundary edges are U/R
None fit - One fit - Both fit
THREE-DIRECTIONAL 😊

{U,D,R}-LEMMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
Both boundary edges are U/R
None fit - One fit - Both fit
\{U,D,R\}-LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
Both boundary edges are U/R
None fit - One fit - Both fit
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
Both boundary edges are U/R
None fit - One fit - Both fit
\{\text{U,D,R}\}\text{-LEMMA}

A \{\text{U,D,R}\}-path admits a direction-consistent embedding on a convex point set

“\text{PROOF}”

Both boundary edges are \text{U/R}

None fit - One fit - Both fit

Apply “strip-convex” Lemma
\{U,D,R\}-LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
Both boundary edges are U/R
None fit - One fit - Both fit
Apply “strip-convex” Lemma
Sort by y-coordinate
\textbf{{U,D,R}}-\textbf{LEMMA}

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

\textbf{“PROOF”}

Both boundary edges are U/R
None fit - One fit - Both fit
\{U,D,R\}-LEMMA

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”

Both boundary edges are U/R

None fit - One fit - Both fit
{U,D,R}\-LEMMA

A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”

Both boundary edges are U/R
None fit - One fit - Both fit
\{U,D,R\}-LEMMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
Both boundary edges are U/R
None fit - One fit - Both fit
\{U,D,R\}-LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
Both boundary edges are U/R
None fit - One fit - Both fit
\{U,D,R\}-LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
Both boundary edges are U/R
None fit - One fit - Both fit
Apply “strip-convex” Lemma
\{U,D,R\}-LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
Both boundary edges are U/R
None fit - One fit - Both fit
Apply “strip-convex” Lemma
Sort by y-coordinate
\{U,D,R\}-LEMMA
A \{U,D,R\}-path admits a direction-consistent embedding on a convex point set*

“PROOF”
Both boundary edges are U/R
None fit - One fit - Both fit
Apply “strip-convex” Lemma
Sort by y-coordinate
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set.
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“PROOF”
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“PROOF”
General convex point set and \{U,D,R\}-path
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“PROOF”
General convex point set and \{U,D,R\}-path
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“PROOF”
General convex point set and \{U,D,R\}-path
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“PROOF”
General convex point set and \{U,D,R\}-path

Mirror the point set and the path.
THREE-DIRECTIONAL 😊

THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“PROOF”
General convex point set and \{U,D,R\}-path
Mirror the point set and the path.
Get a \{U,D,L\}-path

Embedding 4-directional path on a convex point set
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“PROOF”
General convex point set and \{U,D,R\}-path
Mirror the point set and the path.
Get a \{U,D,L\}-path
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“PROOF”
General convex point set and \{U,D,R\}-path

Mirror the point set and the path.

Get a \{U,D,L\}-path
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“PROOF”
General convex point set and \{U,D,R\}-path

Mirror the point set and the path.

Get a \{U,D,L\}-path

Reverce the path and the labels, get a \{U,D,R\}-path
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“PROOF”
General convex point set and \{U,D,R\}-path
Mirror the point set and the path.
Get a \{U,D,L\}-path
Reverce the path and the labels, get a \{U,D,R\}-path
Apply the \{U,D,R\}-Lemma
THEOREM
Any three-directional path admits a direction-consistent embedding on any convex point set

“PROOF”
General convex point set and \{U,D,R\}-path

Mirror the point set and the path.

Get a \{U,D,L\}-path

Reverse the path and the labels, get a \{U,D,R\}-path

Apply the \{U,D,R\}-Lemma

Treat \{U,D,L\}, \{R,L,U\} and \{L,R,D\}-paths similarly
EMBEDDING 4-DIRECTIONAL PATHS ON CONVEX POINT SETS

RESULTS

Not always possible for four directions

Always possible for three directions

Can be decided in $O(n^2)$ time for four directions.
EMBEDDING 4-DIRECTIONAL PATHS ON CONVEX POINT SETS

RESULTS
Not always possible for four directions
Always possible for three directions
Can be decided in $O(n^2)$ time for four directions.

OPEN PROBLEMS
Does every oriented path admit an upward planar embedding on every point set?

Embedding 4-directional path on a convex point set
EMBEDDING 4-DIRECTIONAL PATHS ON CONVEX POINT SETS

RESULTS
Not always possible for four directions
Always possible for three directions
Can be decided in $O(n^2)$ time for four directions.

OPEN PROBLEMS
Does every oriented path admit an upward planar embedding on every point set?

If yes, can we do the construction in polynomial time? If no, what is the complexity of the problem?
CONCLUSION

EMBEDDING 4-DIRECTIONAL PATHS ON CONVEX POINT SETS

RESULTS

Not always possible for four directions

Always possible for three directions

Can be decided in $O(n^2)$ time for four directions.

OPEN PROBLEMS

Does every oriented path admit an upward planar embedding on every point set?

If yes, can we do the construction in polynomial time? If no, what is the complexity of the problem?

Are the four-directional planar drawings interesting by themselves? (no point set given)
EMBEDDING 4-DIRECTIONAL PATHS ON CONVEX POINT SETS

RESULTS

Not always possible for four directions
Always possible for three directions
Can be decided in $O(n^2)$ time for four directions.

OPEN PROBLEMS

Does every oriented path admit an upward planar embedding on every point set?

If yes, can we do the construction in polynomial time? If no, what is the complexity of the problem?

Are the four-directional planar drawings interesting by themselves? (no point set given)

THANK YOU!